CAPACITIES*

BY

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DIELECTRICAL

As the seat of energy of an electrical field is in the space outside of the charged bodies we will consider the shape and concentration of the field only, but not that of the body itself. This distinction is necessary because capacities are usually attributed to the bodies charged, whereas the energy is excluded from that space which is occupied by the body. Considering the space between two charged bodies as the only seat of energy, the expression "charged body" is best replaced by "terminal surface" of the field.

Comparing geometrically similar elements of two geometrically similar fields, the elementary capacities are proportional to lineal dimensions. (See Figure 1.)

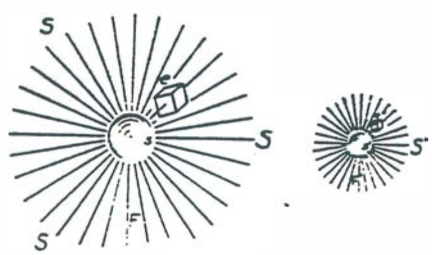

FIGURE 1

Extending this law over the entire field by the integrating process, we find that geometrically similar fields have capacities proportional to the lineal dimensions of the terminal surfaces. It is to be expected, therefore, that capacities expressed in dimensions of terminal surfaces should be of lineal dimensions.

That the capacity is by no means a function of the volume of the field or of the terminal body may be easily seen from Figure 2 where a field element is increased to double the volume by adding

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volume in the direction of the field lines and in a direction perpendicular to the lines. In the first case the capacity has been decreased whereas in the latter case increased, altho in both cases the volumetric increase is the same.

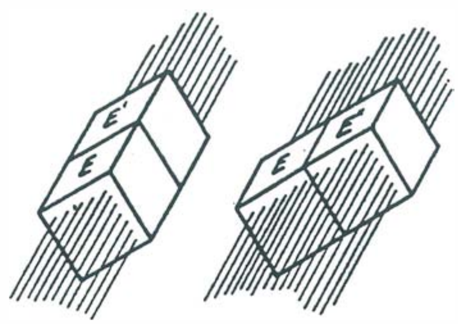

FIGURE 2

It is seen, therefore, that instead of being dependent on the volume, the capacity is rather a function of lineal dimension and therefore the maximum lineal dimension predominates.

An interesting example of this predominating lineal dimension or "maximum reach" is given by the composite capacity of two wires joining at one end under various angles, as shown in Figure 3.

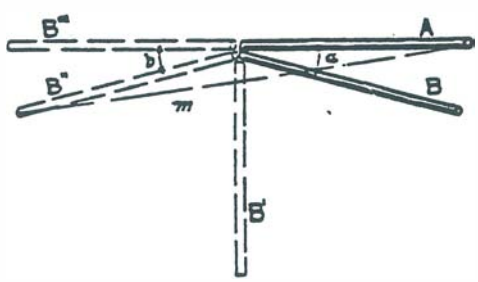

FIGURE 3

When the angle is small the composite capacity is practically the same as that of the single wire, since the addition of the second wire has not increased the maximum reach. If the second wire *B* be joined to *A* at an angle of 180 degrees, which means in straight continuation of wire *A* the total capacity has oubled, as the maximum reach now is twice that of the single wire. We notice also that by deviating wire B slightly from the traight continuation of wire A, the maximum reach of the system is not materially altered, from which one may correctly conclude that turning the wire B thru an appreciable angle b does not materially change the capacity of the system. On the other hand a great change of maximum reach is produced by variations of the angle when the two wires are approximately perpendicular, and in fact the capacity of the total structure is most sensitive to changes of angle between the two elements at about 90 degrees.

In Figure 4, I have given a table of capacities per centimeter of the greater lineal dimension of the different configurations.

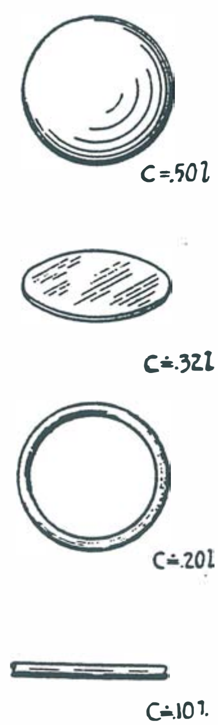

Figure 4

In Figure 5 the wire AB is assumed to be moved by the variable abscissae x, thereby generating a conducting sheet S.

It is instructive to follow the variation of the capacity C_x.

At $x=0$ the capacity is that of the wire C_{ab}; as long as x is small the capacity is practically constant because the width of the sheet is small compared to the length AB and a change of x does not involve a change of the predominating lineal dimension; however, as x increases and finally becomes greater than AB, it assumes the part of the predominating dimension, and, indeed, the graph shows the capacity then to be proportional to x.

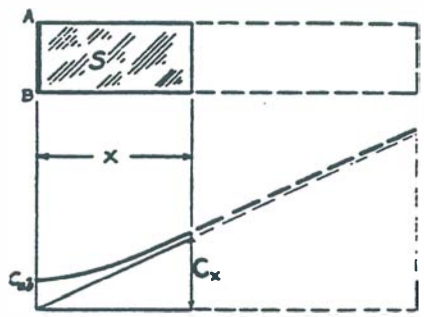

FIGURE 5

Comparing the capacities of a sphere and of a wire, it is found that the capacity of the sphere is only three or four times as great as the capacity of the wire in spite of the million times greater volume.

I have spoken of the capacities of a wire and of other bodies instead of the capacity of the field simply because I do not wish to distract attention from the familiar conceptions. Let me analyze the field shown in Figure 6, having two concentric spheres as terminal surfaces, and defining as "volumetric energy density" the energy contained in one cubic centimeter. As the energy of a field element is made up of the product of potential along the lines of force within that element and of the number of lines traversing it, the energy of a cubic centimeter of electric field is proportional to the square of the field density. Since the field density diminishes as the square of the distance from the center of field, the volumetric energy density diminishes with the fourth power of the distance from the center. The diagram to the left in Figure 6 shows the decrease of volumetric energy density.

Of greater interest than the volumetric energy density is the lineal energy density, which may be defined as the energy contained

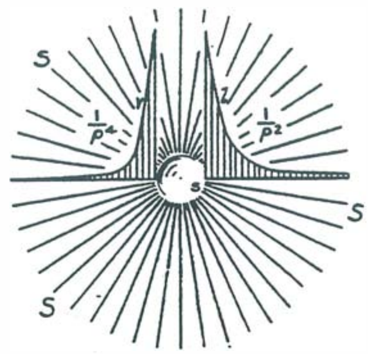

FIGURE 6

in a spherical layer of one centimeter radial thickness; and as the volume of such layer increases with the square of the distance from the center, the law follows from this fact, and from the volumetric energy density law that the lineal energy density decreases inversely as the square of the distance from the center. Such dependence is graphically shown to the right in Figure 6. The shaded surface below this curve represents the total energy of the field and it is easily seen therefrom that the maximum energy of the field is concentrated near the smaller of the two spheres.

I have taken a simple case of a field with spherical terminal surfaces to show that the concentration of energy lies near the smaller terminal surface. Similar considerations can be applied when substituting for this field radiating three-dimensionally, a field of bi-dimensional radiation (as that occurring in the case of long cylindrical terminal surfaces); where, as in this instance, the bulk of the energy of the field is to be found near the smaller one of the two terminal surfaces.

In Figure 7, I have shown a field with concentric terminal surfaces (either spherical or cylindrical), and have increased the scope of the field by reducing the size of the smaller terminal surface without, however, changing either the total number of field lines or the larger terminal surface. As the lineal energy density is very great near the smaller terminal surface, such addition of the field at that point must have materially increased the energy of the field and the change in capacity to be expected should be considerable. In fact, a considerable change in capacity of a sphere is obtained by a change of its diameter.

If, in Figure 7 the larger terminal surface alone is changed, even materially, the total energy of the field will be increased very slightly only; due to the fact, as we have seen, that the energy density near the larger terminal surface is very small. Such a small change in energy corresponds to only a small change in the capacity of the field, from which we conclude:

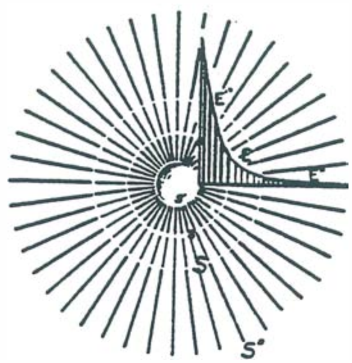

FIGURE 7

In a field having two terminal surfaces of greatly different size, a change of the smaller surface produces a great change in capacity, whereas a change of the larger terminal surface affects the capacity of the field only very slightly. The capacity of a field is, therefore, almost entirely determined by the shape of the smaller terminal surface.

That is why we may with correctness speak of the capacity of a sphere, or any other body, without mentioning the size and shape of the other terminal surface, as long as the assumption is correct that such other terminal surface is of greatly larger dimensions.

It may not be amiss to call your attention to the fact that the increase of field energy as illustrated in Figure 7 is accompanied by a decrease in capacity. This relation may easily be deduced from physical considerations, as well as from consideration of the mathematical expression for the capacity

$$C = \frac{\phi^2}{32\pi^2 W} \qquad \text{where } \phi = \text{total field lines}, \quad W = \text{energy},$$

wherein the capacity is expressed as a property of the field alone. I am tempted to introduce here the reciprocal value of capacity and apply to it the term "stiffness of the field," as an increase of energy would be followed by an increase of stiffness. I am, however, loath to mar any additional insight which may be gained from these explanations by deviation from so familiar a term as capacity.

For a better conception of the slight change of capacity caused by a considerable increase of the larger terminal surface, I refer to Figure 7, where the difference of capacity is only 1 per cent in spite of the diameter of the larger terminal surface being increased 100 per cent. It appears, therefore, that that part of the capacity of an antenna which is due to the flat top is not materially changed by its height above ground.

While considering the capacity of a flat top antenna to ground, it must have occurred to many engineers, as it did to me, that the statement to be found in many text books on electrostatics is rather misleading: "That the free capacity of a body considered alone in space must not be confounded with the capacity the body may have against another body considered as a plate condenser." This statement is quite erroneous. As the strength and direction in any point of a field is of single and definite value, only one electric field can exist in a given space at a given moment, and, therefore, only one value of capacity. It is incorrect, therefore, to distinguish between free capacity and condenser capacity. This clarifying statement is deemed advisable, or at least permissible, in view of the quoted errors.

By speaking of the capacity of the field instead of that of the body, no such erroneous thought is possible, and it is clear that by free capacity of a body is meant the capacity of the field whose smaller terminal surface is the given body and whose larger terminal surface is one of vastly greater dimensions. It is not essential that this greater terminal surface be located at infinite distance, because of the fact that even if construed as of ten times the lineal dimensions of the small surface the change caused by removing it to an infinite distance would result in a change in capacity of not more than one-tenth of 1 per cent.

At a time when I had not realized the singly determined value of a field capacity, I considered a comparison between free and plate capacity as shown in Figure 8, wherein to an upper disc (of which the free capacity is $\frac{2}{\pi}r$), was added another lower disc, thereby forming a plate condenser. The problem arose in my mind to determine the distance of separation of the two plates so that the plate capacity would equal the free capacity of the single disc. From the well-known formulas for the disc

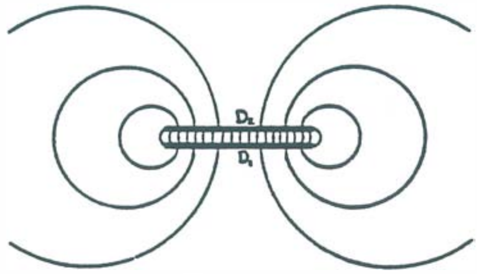

FIGURE 8

capacity and plate capacity, it would appear that the two were equal at a distance equal to $d=\frac{\pi}{8}r$, and I must confess that I had quite a struggle to decide whether in speaking of the capacity of the upper plate I would not have to add the two capacities. While such a mistake need hardly be called to the attention of the majority of engineers, I do not hesitate to make mention of it for the benefit of even the few students who might gain therefrom.

The advent of the aeroplane has opened another field, for radio communication. Whereas in the static field of an antenna, one terminal surface is artificial and the other provided by the surrounding ground, both terminal surfaces in an aeroplane outfit have to be artificial and are, therefore, open to design. The question arises in such a radio oscillator as to how much may be gained in energy for each single charge by increasing that one of the two terminal surfaces which consists of a dropped wire. The arrangement is shown in Figure 9. It is evident that

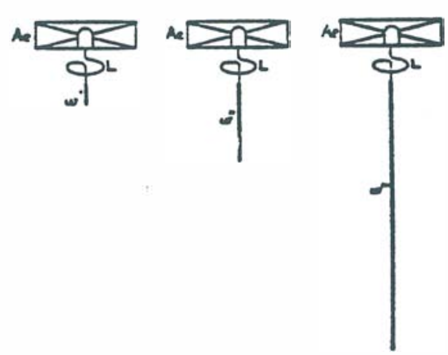

FIGURE 9

as long as the dropped wire is of smaller dimensions than the electrostatic counterpoise provided on the aeroplane, an increase in length of such dropped wire will materially increase the capacity of the field and, therefore, the energy per charge (as we may conclude by analogy from Figure 7). As soon, however, as the dropped wire is materially longer than the conductor on the aeroplane it assumes the role of the larger terminal surface of the field, and any further increase of its length will not materially contribute to an increase of electrostatic capacity nor of the energy per unit charge.

Figure 10 shows the function of the volumetric and lineal energy density in a field whose smaller terminal surface is a long cylinder. Such a field, radiating bi-dimensionally only, shows an energy concentration not so accentuated as that found in the

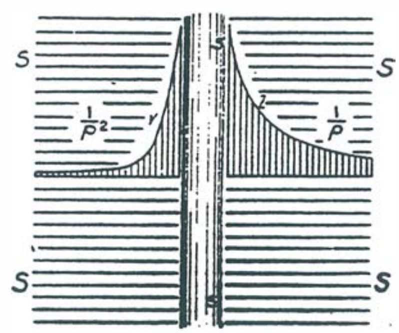

FIGURE 10

tri-dimensionally radiating field; but considering the larger terminal surface of a diameter ten times that of the smaller surface, the capacity would only be changed 1 per cent by increasing the larger terminal surface infinitely.

In all cases, therefore, where the larger terminal surface does not come closer at any point than (say) ten times the corresponding dimension of the smaller terminal surface, we need not be concerned with the actual shape of the larger terminal surface when we determine the seat of energy, the capacity and the configuration of the field lines emanating from the smaller surface. It will be seen, therefore, that from the flat top of an antenna, lines emanate almost symmetrically both upwards and downwards as though the larger terminal surface were one surrounding the antenna symmetrically on all sides, in spite of the fact that the ground is located entirely at the bottom of the antenna. This is clearly illustrated in Figure 11.

By integrating the lineal energy density of a three-dimensionally radiating field between the radius of the smaller sphere

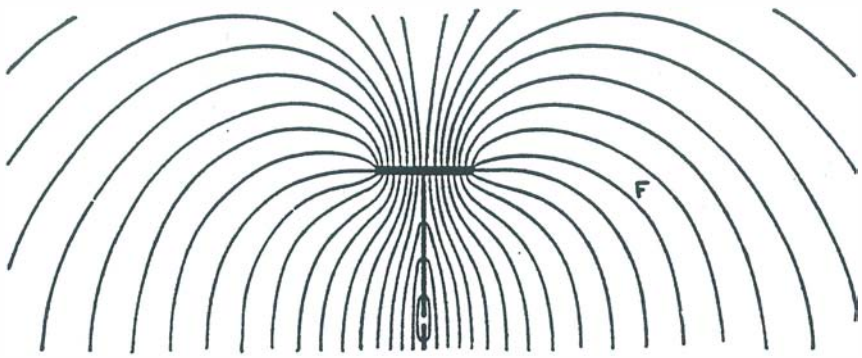

FIGURE 11

and that of the larger sphere, we can find the energy of such a field; whereby the capacity is determined. The lineal energy density follows the law of $\frac{1}{r^2}$, and its integral is proportional to $\frac{1}{r}$; and consequently the capacity of the field varies as r.

We have deduced, therefore, the capacity of a sphere from properties of the field alone, considering the sphere as a terminal surface only.

In deducing similarly the capacity of the wire from properties of the field alone, we have to start with the bi-dimensionally radiating field the lineal energy density of which follows the law $\frac{1}{r}$ as we have seen. The integral of such function is of logarithmic nature, as indeed is the capacity of the wire.

I wish to call your attention to the fact that in a sphere segments of the same projected axial length contribute equally to the capacity of the sphere, as shown in Figure 12.

If a charge were made to enter a sphere and traverse the sphere in the direction of a diameter, the sphere as a conductor would behave like a straight piece of wire of uniform lineal capacity. This fact was first recognized, to my knowledge, by

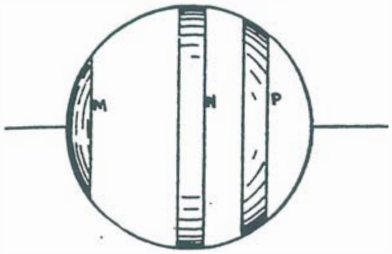

FIGURE 12

Mr. Nikola Tesla, and I expect to come back to the behavior of a sphere as a conductor of radio frequency currents at some later date.

The study of capacities of composite bodies is most instructive and conducive to a clear conception of capacity. Let, as in Figure 13, a number of small spheres of radius be so arranged as to cover completely the surface of the larger sphere, the radius R of which be 100. If each one of the

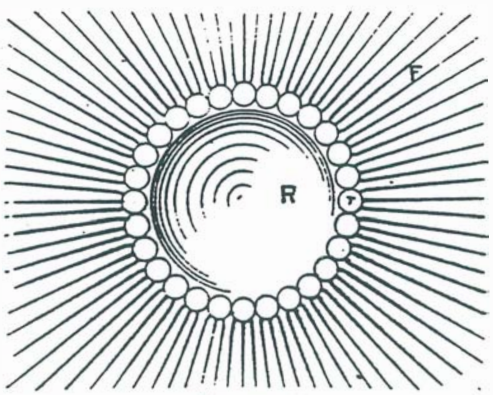

FIGURE 13

31,400 smaller spheres could be counted at its full value of capacity, the capacity of the composite body would be 31,400; as a matter of fact, however, it is not more than radius R of the larger sphere, that is 100. Indeed, the configuration of the electric field F could not have changed materially by the arrangement of the small spheres, and the capacity clearly presents itself as a property of the configuration of the field lying outside of the enveloping surface of the composite structure.

Capacity may play a part in the conduction of electricity thru liquids and gases. Let us assume a series of spheres in lineal arrangement as shown on Figure 14.

As long as the distance between the spheres is great compared to the diameter of the spheres, each sphere will retain its full capacity as given by its radius. By decreasing the distance

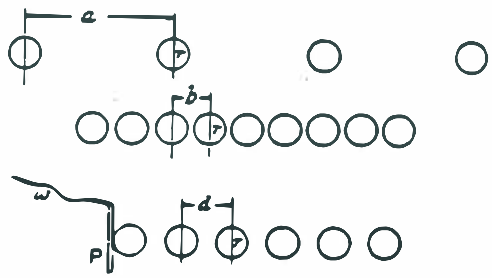

FIGURE 14

between spheres the individual capacities of the spheres decrease, because of the negative capacity coefficients. If such approximation be carried to the point of contact between the spheres, the capacity of each individual sphere would be reduced to approximately $\frac{1}{\varepsilon}$ of the original capacity. If such a row of spheres were conceived as freely movable, so as to enable each sphere to make contact with a plate P, which is kept charged to a certain potential, then the charges carried away by the spheres after contact with the plate would be proportional to the full capacity of each sphere as long as the spheres are far apart, and would be only $\frac{1}{\varepsilon} = \frac{1}{2.718}$ *th* part of such maximum charge when the spheres are in contact. As we assumed the plate P to be maintained at a certain potential by an outside source of electricity, the convection current represented by the departing charges of the spheres would vary approximately in a ratio of 2.71 to 1:

In the passage of electricity thru an electrolyte, the molecular conductivity has been found to be the same for all electrolytes, and varying only with the concentration of the solution; the molecular conductivity being approximately 2.5 times as great in the very dilute solution as in the concentrated solution.

I wish to call your attention to the striking similarity between the ratio of conductivity experimentally determined in electrolytes of small and large concentration and the ratio of conductivity of the row of spheres where the spheres are far apart or close together. I do not pretend at this moment that a

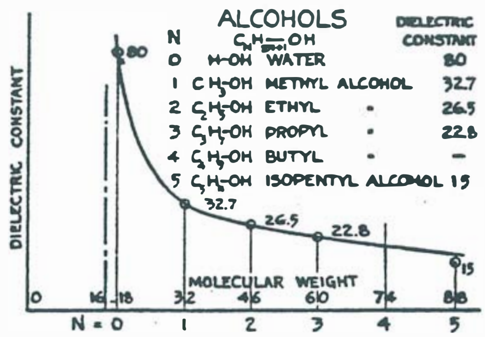

Figure 15

plausible modification of the theory of conduction thru electrolytes and gases can be based on such a coincidence; and in fact, assumptions would have to be made. For example, a lineal arrangement of the ions in the direction of the static field impressed on the electrolyte or on the gas must be assumed.

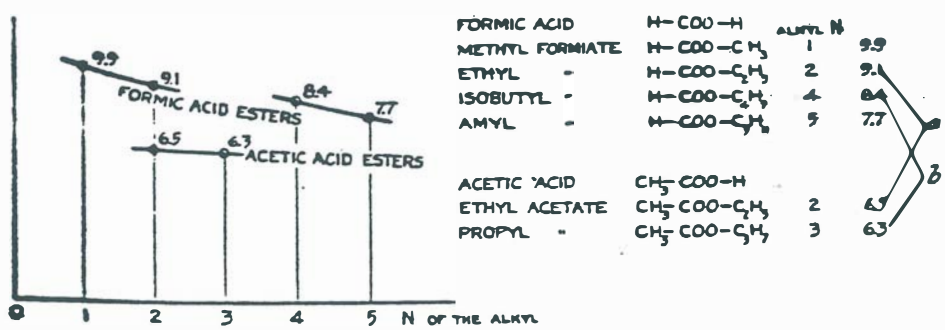

Figure 16

But the fact that such ratio in the case of the spheres is deduced from geometrical considerations alone, coupled with the fact that in electrolytes the same ratio follows from purely geometrical considerations, is sufficient to warrant further thought. I do not hesitate to bring this interesting coincidence to your knowledge, with the hope that other physicists may carry on investigations in the same direction. I have said that the molecular conductivity of electrolytes arose from geometrical considerations only, and I think it advisable to call your attention to the foundation of such a statement. While it is true that the conductivity of different electrolytes varies considerably, it has been found that the molecular conductivity is the same for all electrolytes. The similar behavior, of the same number of molecules, independently of the weight of the molecule, therefore reduces the phenomenon to a purely geometric basis.

SUMMARY: Considering that electrostatic energy is actually in the space surrounding a charged body, the latter is called a "terminal surface." It is shown that capacity is predominantly a function of the ***maximum*** lineal dimension of the terminal surface. The volumetric and lineal energy densities in the field are defined and studied in a number of cases. It is proven that the capacity between two terminal surfaces is greatly affected by changing the lineal dimensions of the ***smaller*** terminal surface, but not so for changes of the larger. Certain current errors in connection with "mutual capacity" are considered.

The practical applications to a radio antenna and to aeroplane counterpoises are given.

When a charge traverses a sphere, entering parallel to a diameter, the sphere behaves as a conductor of uniform lineal capacity.

Applications of the theoretical considerations are also given in connection with the conductivity of concentrated and dilute electrolytes.